A Quotient Sequence from Arithmetic and Geometric Progressions

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Introduction

It is known that $\{a_n\}$ is an infinite arithmetic sequence with the first term being a and a tolerance of 2. $\{b_n\}$ is an infinite arithmetic sequence with the first term being 1 and a common ratio of 2. Let' s call it $\lambda_n = \dfrac{a_1 + a_2 + \cdots + a_n}{b_n}$. The following four conclusions are given:

①When $0 < a < 2$, there is $\lambda_1 < \lambda_2 < \lambda_3$;

② There exists $a \in \mathbb{R}$, so that the first 2025 items of $\{\lambda_n\}$ are monotonically increasing sequences;

③For any $a > 0$, $\{\lambda_n\}$ are monotonically decreasing sequences from the third item onwards;

④ If and only if $a = 2$, there exists $k \in \mathbb{N}^*$ such that $\lambda_k = \lambda_{k+1}$.

The serial numbers of all correct conclusions are ___ .

First, remember $S_n = \sum_{i=1}^{n} a_i$ and calculate the first three terms of S_n, b_n, λ_n:

n	S_n	b_n	λ_n
1	a	1	a
2	$2a+2$	2	$a+1$
3	$3a+6$	4	$\frac{3a+6}{4}$

To satisfy $\lambda_1 < \lambda_2 < \lambda_3$, as long as:

$\frac{3a+6}{4} > a + 1$ is sufficient, which is equivalent to:

$a < 2$, so ① is correct.

To let $\lambda_{n+1} > \lambda_n$, it is equivalent to:

$S_{n+1} > 2S_n$, that is:

$$a_{n+1} > S_n$$

Stronger, just:

$a_n < 0(n = 1, 2, 3, ..., 2024)$ can satisfy $S_n < S_{n-1} < ... < S_1 = a < a_{n+1}$

Furthermore, we study the necessary and sufficient conditions for ②:

$$a_{n+1} = a + 2n > S_n = na + \frac{n(n-1)}{2} \times 2 = na + n(n-1)(n = 1, 2, .., 2024)$$

$n = 1$, obviously there is: $a_2 = a + 2 > a_1 = a$

※: $a < \frac{-n^2+3n}{n-1} = -n + 2 + \frac{2}{n-1}(n = 2, 3, .., 2024)$

$u_n = -n + 2 + \frac{2}{n-1} = 1 - (n - 1 - \frac{2}{n-1})(n = 2, 3, ..., 2024)$ is a decreasing sequence.

So ② is equivalent to: $a < u_{2024} = -2022 + \frac{2}{2023}$, ② is correct.

Look again ③:

Based on the analysis of ②, $a > 0 = u_3$, that is, $n > 3$ satisfies $\lambda_{n+1} < \lambda_n$, and ③ is correct.

Last look ④:

As long as $a = u_n$ is taken, $\lambda_n = \lambda_{n+1}$ and ④ errors can be satisfied.

In summary, choose ①②③.

Practice

(End of the first semester of the second grade of Beijing Daxing High School in 2026) (10) It is known that $\{a_n\}$ is an infinite arithmetic sequence in which all terms are not zero, and the tolerance is d. Let $c_n = \frac{a_1 + a_2 + \cdots + a_n}{a_n}$, if $a_3 \cdot d < 0$, then the sequence $\{c_n\}$

(A) There is a maximum term but no minimum term

(B) There is no maximum term and no minimum term

(C) There is a maximum term and a minimum term

(D) There is no maximum term, but there is a minimum term

Let $a_1 = a,\ a_n = a + (n-1)d$; then $a_3 \cdot d = d(a + 2d) < 0$.

$$c_n = \frac{S_n}{a_n} = \frac{n(a_1 + a_n)}{2a_n} = \frac{n}{2}(1 + \frac{a_1}{a_n}) = \frac{n}{2}(1 + \frac{a}{a + (n-1)d}) = \frac{n}{2}(1 + \frac{1}{1 + (n-1)\frac{d}{a}})$$

Also $\frac{d}{a}(1+2\frac{d}{a})<0$, so $\frac{d}{a}\in(-\frac{1}{2},0)$.

When n is sufficiently large, $c_n/(\frac{n}{2})\to 1, c_n\to+\infty$ has no maximum term.

Obviously the sign of c_n is determined by $1+\frac{1}{1+(n-1)\frac{d}{a}}$. If the terms less than 0 in c_n are finite, there must be the smallest c_n.

$$
\begin{gathered}
1+\frac{1}{1+(n-1)\frac{d}{a}}<0 \\
\Leftrightarrow 1+(n-1)\frac{d}{a}\in(-1,0) \\
\Leftrightarrow (n-1)\in(0,\frac{-2}{\frac{d}{a}})
\end{gathered}
$$

Among them, $\frac{-2}{\frac{d}{a}}>4$, such n has only limited possibilities, so c_n has a minimum term.

PS: Grok didn’ t solve this question, but DeepSeek did it, and Musk came out and got beaten)

Conclusion

> For the extreme value problem of a sequence, we must not only look at the trend, but also the "turning point". Changes in sign often reveal the existence of extreme values better than monotonic intervals. When the trend at infinity of c_n is established, the extreme value is hidden between the sign interleaving of the finite terms. If you can grasp this point, it shows that your understanding of the sequence structure is quite profound. As for Grok, it may be good at massive pattern matching, but where mathematical insight is really needed, it's still a bit "smart". If you continue to think like this, the final question on the number sequence in the college entrance examination will be nothing more than a paper tiger.